

Self-similar community structure in organisations

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The formal chart of an organisation is designed to handle routine and easily anticipated problems, but unexpected situations arise which require the formation of new ties so that the corresponding extra tasks can be properly accomplished. The characterisation of the structure of such *informal networks* behind the formal chart is a key element for successful management. We analyse the complex e-mail network of a real organisation with about 1,700 employees and determine its community structure. Our results reveal the emergence of self-similar properties that suggest that some universal mechanism could be the underlying driving force in the formation and evolution of informal networks in organisations, as happens in other self-organised complex systems.

Although the formal chart of an organisation is intended to prescribe how employees interact, ties between individuals arise for personal, political, and cultural reasons [1]. The understanding of the formation and structure of such informal networks are key elements for successful management [1, 2, 3]. The traditional way of investigating informal networks within organisations consists of conducting surveys using employee questionnaires. However, employees answers often contain subjective elements such as “political” motives and the worry about offending colleagues. Another significant limitation of the questionnaire based analysis is that time and effort costs make it prohibitively expensive to map the entire network even for medium sized organisations. The rapid development of electronic communications provides a powerful alternative to the traditional analysis of informal networks. Indeed, the exchange of e-mails between individuals in organisations reveals how people interact and allows mapping the informal network in a non-intrusive, objective, and quantitative way.

We surmise that the exchange of e-mails between individuals in organisations reveals how people interact [4, 5] and therefore provides a map of the real network structure behind the formal chart. We analyse the complex e-mail network of a real organisation with about 1,700 employees and determine its community structure [6, 7, 8, 9]. Our results reveal the emergence of self-similar properties that suggest that some universal mechanism could be the underlying driving force in the formation and evolution of informal networks in organisations, as happens in other self-organised complex systems [10].

Every time that an e-mail is sent, the addresses of the sender and the receiver are routinely registered in a server. Therefore, an *e-mail network* can be built regarding each e-mail address as a node and linking two nodes if there is an e-mail communication between them. In particular, we take as a case study the e-mail network of University Rovira i Virgili (URV) in Tarragona, Spain, containing around 1700 users (Fig. 1). Bulk e-mails provide little or no information about how individuals or teams collaborate and, once they are removed, the connectivity distribution of the e-mail network is exponential,

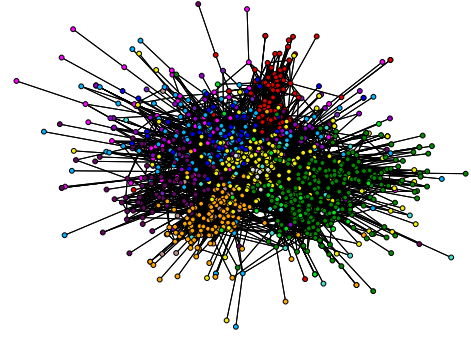


FIG. 1: The e-mail network of URV. The network comprises approximately 1700 users, including faculty, researchers, technicians, managers, administrators, and graduate students. We consider e-mails exchanged between university addresses during the first three months of 2002. Each individual is represented by a node, with two individuals (A and B) being connected if A has sent an e-mail to B and B has also sent an e-mail to A. Bulk e-mails provide little or no information about how individuals or teams collaborate. To minimise their effect: (i) we eliminate e-mails that are sent to more than 50 different recipients and (ii) we disregard links that are unidirectional, that is we consider only e-mails that represent a real communication link, where e-mails flow in both directions. With these two restrictions, the network is undirected and is formed by a main component comprising 1133 nodes and many isolated nodes or pairs of nodes. These little islands are not plotted to keep the figure as simple as possible. The colour of each node identifies an individual's affiliation to a specific centre within the university.

$P(k) \propto \exp(-k/k^*)$ for $k \geq 2$, with $k^* = 9.2$. This result is in contrast with recent findings indicating that some technology based social networks—such as rough e-mail networks [4], the Instant Messaging Network [11] or the PGP encryption network [12]—show heavily skewed degree distributions, but is consistent with the proposal of Amaral and coworkers that the truncation of the scale-free behaviour in real world networks is due to the existence of limitations or costs in the establishment of connections [6]. Indeed, it seems plausible that there are costs to maintaining active social acquaintances and therefore active communications. However, it is relatively easy to keep many *electronic* acquaintances *open*, although

same colour. This shows that the identification of communities is successful, despite the complexity of the network. Second, the branching structure is far from simple. Indeed, each branch is formed, in general, by a system of nested smaller subbranches that give rise to a complicated structure that visually resembles some self-similar systems in nature such as river networks [13] or diffusion-limited aggregates [14]. For comparison, we also show the tree generated by the GN algorithm from a random graph of the same size and average connectivity as the e-mail network (Fig. 3c). In contrast to the tree for the URV e-mail network, the branching structure is almost trivial with most of the branches containing only 1 or 2 nodes. This is the expected result for a network that do not have any sort of community structure.

Once the binary tree has been obtained, we look for a quantitative characterisation of the community structure. First we consider the cumulative community size distribution, $P(s)$, i.e. the probability of a community having a size larger or equal to s . Fig. 4a shows how to compute this probability, and the resulting distribution for the e-mail network is presented in Fig. 4d. The distribution is heavily skewed, following a power law behaviour $P(s) \propto s^{-\alpha}$ with $\alpha = 0.48$ between $s = 2$ and $s \approx 100$. Beyond this value, the distribution shows a sharp decay and, at $s \approx 1000$, a cutoff that corresponds to the size of the system. The power law of the community size distribution suggests that there is no characteristic community size in the network (up to $s \approx 100$). To rule out the possibility that this behaviour is due to the community identification algorithm, we also consider the community size distribution for a random graph with the same size and average connectivity as the e-mail network.

The characterisation of the community binary tree using the cumulative size distribution has its analogy in the river network literature [13, 15, 16]. The equivalent measure is the distribution of drainage areas, that represents the amount of water that is generated upstream of a given point (see Fig. 4b). The similitude between the community size distribution of the current e-mail network in Fig. 4d and the area distribution of the Fella river network in Italy reported in Fig. 2 of Ref. [16] is striking. The exponent $\alpha = 0.45$ for the power law region of this river and the average exponent for several rivers $\alpha_{river} = 0.43 \pm 0.03$ respectively reported by [16] and [15], are very close to the current $\alpha = 0.48$. Moreover, the behaviour shown in Fig. 4d with first a sharp decay and then a final cutoff is also shared by river networks, which are known to evolve to a state where the total energy expenditure is minimised [15, 17, 18]. The possibility that communities within organisations might also spontaneously self-organise into a form in which some quantity is optimised is very appealing and deserves further investigation.

To further understand this point, it is pertinent to ask the question: does the similarity between community trees in organisations and river networks arise just by chance or are there other emergent properties shared by both? To answer this question we consider a standard measure for categorising binary trees: the Horton-Strahler (HS) index, originally intro-

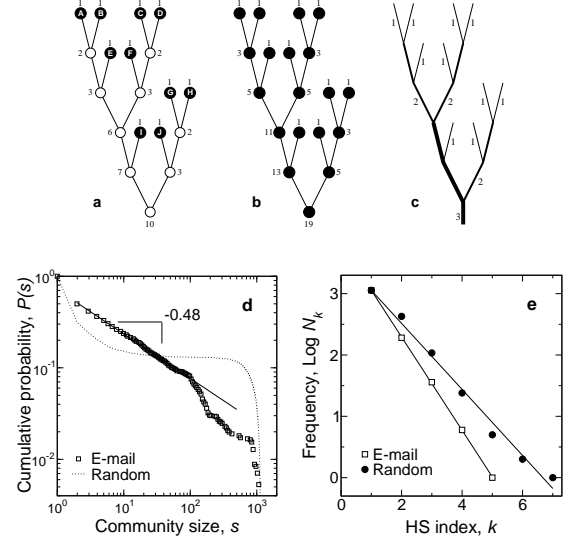


FIG. 4: Self-similarity in the community structure. **a**, Calculation of the community size distribution for a binary tree generated by the community identification algorithm. Black nodes represent the actual nodes of the original graph while white nodes are just graphical representations of communities that arise as a result of the splitting procedure. Nodes *A* and *B* belong to a community of size 2, and together with *E* form a community of size 3. Similarly, *C*, *D* and *F* form another community of size 3. These two groups together form a higher level community of size 6. Following up to higher and higher levels, the community structure can be regarded as the set of nested groups. The size, s_i , of a community i is just the summation of the sizes of its two offspring j_1 and j_2 : $s_i = s_{j_1} + s_{j_2}$. In this case there are three communities of size 2, three communities of size 3, one community of size 6, one community of size 7, and one community of size 10. Note that a single node belongs to different communities at different levels. **b**, Calculation of the drainage area distribution for a river network. The drainage area of a given point is the number of nodes upstream of it plus one. For a point i with offspring j_1 and j_2 , $s_i = s_{j_1} + s_{j_2} + 1$. **c**, Calculation of the Horton-Strahler index. The index of a branch changes when it meets a branch with higher index, or when it meets a branch with the same value and both of them join forming a branch with higher index. In this case, there are 10 branches with index 1, 3 branches with index 2, and 1 branch with index 3. **d**, The distribution of community sizes, $P(s)$, showing a power law region with the exponent -0.48, followed by a sharp decrease at $s \approx 100$ and a cutoff corresponding to the size of the system at $s \approx 1000$. The distribution of community sizes in a random network is shown with a dotted line for comparison. **e**, The number of branches with HS index i , as a function of i . From the definition of the branching ratio, it is straightforward to show that, when topological self-similarity holds, $N_i = N_1/B^{i-1}$. A fitting of this function to the points obtained for the e-mail community tree yields excellent agreement with $B = 5.76$. A much worse agreement is obtained for the community tree corresponding to the random network, with B_i fluctuating around 3.46.

duced for the study of river networks by Horton [19], and later refined by Strahler [20]. Consider the binary tree depicted in Fig. 4c. The leaves of the tree are assigned a HS index $i = 1$. For any other branch that ramifies into two branches with HS

indices i_1 and i_2 , the index is calculated as follows:

$$i = \begin{cases} i_1 + 1 & \text{if } i_1 = i_2, \\ \max(i_1, i_2) & \text{if } i_1 \neq i_2. \end{cases}$$

Note that the index of a branch changes when it meets a branch with higher index, or when it meets a branch with the same value and both of them join forming a branch with higher index. In terms of communities, the interpretation of the HS index is the following. The index of a community changes when it joins a community of the same index. Consider, for instance, the lowest levels: individuals ($i = 1$) join to form a group (or team, with $i = 2$), which in turn will join other groups to form a *second level* group (or department, $i = 3$). Therefore, the index reflects the *level* of aggregation of communities. The number of branches N_i with index i can be determined once the HS index of each branch is known. The bifurcation ratios B_i are then defined by $B_i = N_i/N_{i+1}$ (by definition $B_i \geq 2$).

When $B_i \approx B$ for all i , the structure is said to be topologically self-similar, because the overall tree can be viewed as being comprised of B sub-trees, which in turn are comprised of B smaller sub-trees with similar structures and so forth for all scales. River networks are found to be topologically self-similar with $3 < B < 5$ [14]. We find that the community tree as generated by the process described above is topologically self-similar with $B_i \approx B = 5.76$ (see Fig. 4e). The same analysis for the communities in a random graph shows that topological self-similarity does not hold, since the values B_i are not constant; they fluctuate around a smaller 3.46 value.

The methods presented here open interesting doors regarding the possibility of mapping the informal network of large organisations in a non-intrusive, objective, and quantitative way. Moreover, the emergence of scaling and self-similarity in the community structure, as well as the similarity with river networks, raises important questions about the mechanisms underlying the interactions between individuals within an organisation. Self-similarity is a fingerprint of the replication of the structure at different levels of organisation, and could be the result of the trade-off between the need for cooperation and the physical constraints to establish connections at any organisational level. At the same time, the similitude with river networks suggests that a common principle of optimisation—of flow of information in organisations or of flow of water in rivers—could be the underlying *driving force* in the formation and evolution of informal networks in organisations.

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